Problem Set 1 #2

Logan Courtright

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1 Problem Description

Rubidium and sapphire crystal oscillators are alternatives to quartz crystal oscillators. What are their advantages and disadvantages?

2 Solution

2.1 Rubidium Oscillators

Rubidium is the 37th element on the Periodic Table, represented by the symbol Rb. Due to its electron shell configuration, rubidium has unique properties that lend it to being used in an oscillatory system. Specifically, rubidium has 4 full layers of electrons and a single electron in the 5th layer S orbital, resulting in 4 "inert" layers and 1 "active" layer that is of interest. Each electron possesses both angular momentum and spin, resulting in the electron creating a magnetic field as it orbits the nucleus. The strength of this magnetic field depends primarily on the angular momentum, but is also influenced by the spin, resulting in different energy levels based on different spins. At the S orbital, the electron has access to only spins of $\pm 1/2$. However, the nucleus of the atom also generates a magnetic field and has its own distinct spin. When the magnetic field of the nucleus and electron interact, the energy state of the electron changes based on the relative spins of both parties. As a result, the lone electron has 4 distinct possible energy levels within the S orbital. This distinction is called "hyperfine splitting" and is a key property of rubidium.

Since there is an energy difference between the different hyperfine levels, exposure to energy will cause the orbital to jump up or down a hyperfine level at a very specific frequency, the "hyperfine transition frequency", creating a resonance system that operates as an oscillator under constant energy exposure. Only radiation that has a frequency very close to the transition frequency will cause this jump, causing an oscillator built on this concept to have a very high Q factor. As a result of these properties, extremely precise clocks can be built, using the known transition frequency of rubidium to serve as a time keeping

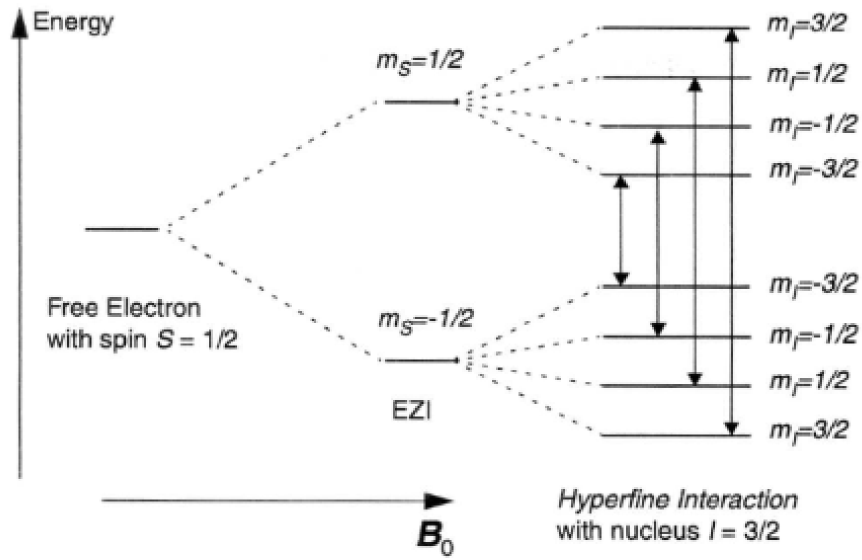

Figure 1: Rubidium Hyperfine Interaction [1]

benchmark. Additionally, these oscillators are also incredibly stable and repeatable due to the precise laws of physics under which they operate.

When compared to quartz crystal oscillators, there are several key advantages and disadvantages of rubidium-based oscillators. Primarily, rubidium oscillators are much more accurate than quartz crystal oscillators; most rubidium oscillators usually are synced to quartz crystal oscillators to keep the quartz crystal oscillators in time. The figure below shows the amount of noise that the oscillator generates when deviations from the transition frequency occur. Additionally, they are much more stable, being less prone to errors from temperature changes or external stress.

However, rubidium oscillators generally require a larger amount of resources to operate correctly. During start up, rubidium oscillators can pull up to 50 W of power and 25 W at steady state, as opposed to the few watts required for most quartz crystal oscillators, resulting in about an order of magnitude more power required. Rubidium oscillators also have longer stabilization times, typically taking about 6 to 9 minutes to reach steady state operation. By comparison, an average quartz crystal oscillator has a start up time of a few hundred microseconds.

Offset(Hz)	Phase Noise(dBc/Hz)
1	-95
10	-135
100	-150
1000	-150

Figure 2: Rubidium Oscillator Phase Noise Diagram [2]

2.2 Sapphire Crystal Oscillators

Sapphire crystal resonators operate under the "Whispering Gallery" principle, where an electromagnetic wave can travel almost completely along the inside of the curvature of a circular sapphire crystal. This principle uses the concept of "total internal reflection", which is the condition that each collision the propagating wave has with the wall results in 100 percent of the wave being contained inside the crystal. In this case, the only loss that occurs to the wave comes from the medium the wave is being contained in, the sapphire in this case. When sapphire is cooled to almost 0 degrees Kelvin the loss that occurs from it is very low. This combination of properties results in a consistent, accurate, and resilient resonator that can be tuned to operate at different frequencies. Additionally, the sapphires used can be lab generated to fit specific specifications, shown by the figure below. This allows for increased customization for different needs. All of these features are improvements over quartz crystal oscillators. However, due to the near absolute zero temperature that sapphire crystals need to obtain these powerful properties, they require large support systems and bulky equipment to maintain a stable temperature. Most sapphire resonator systems use a closed-system cryocooler to as a way to keep the low temperature, but this dependency means that the life cycle of the system is the life cycle of the cryocooler. As a result, sapphire crystal oscillators are best suited for laboratory use where the downsides to such an infrastructure-heavy system can be minimized.

	CSO-1	CSO-2	CSO-3
Frequency ν_0 (GHz)	9.988	9.995	9.987
Material	HEMEX	HEMEX	Kyropoulos
Loaded Q-factor Q_L	1×10^9	350×10^8	400×10^8
Input coupling coeff. β_1	1	1	0.92
Turnover temperature T_0	6.238 K	5.766 K	6.265 K
Injected power	100 μW	300 μW	70 μW

Figure 3: Specifications of Different Sapphire Crystals [3]

3 Works Cited

[1] "Hyperfine Interaction," Eidgenössiche Technische Hochschule Zürich, 2019. https://epr.ethz.ch/education/basic-concepts-of-epr/int–with-nucl–spins/hyperfine-interaction.html

[2] "Selecting Oscillator Technology," Precise Time and Frequency, Inc., 2015. http://www.ptfinc.com/wp-content/uploads/2015/10/Selecting-Oscillator-Technologies.pdf

[3] D. Creedon, Y. Reshitnyk, et al., "High Q-factor sapphire whispering gallery mode microwave resonator at single photon energies and millikelvin temperatures," Applied Physics Letters, 2011. https://web.physics.ucsb.edu/ martinisgroup/papers/Creedon2011.pdf

3.1 Works Cited Uses

Section 2.1 - Completed using [1] and [2]
Section 2.2 - Completed using [3]